HBSU

Dr. Homi Bhabha State University
Mumbai

Proposed M. Sc. (Mathematics) Syllabus

Semester I

With Effect from Academic Year 2023 - 2024

Preamble

Mathematics, on the one hand, has purity and beauty, and on the other, is a contemporary subject whose concepts and methodology are being used by working Physicists, Statisticians, Computer Scientists, Chemists and Biologists.

Postgraduate degree in Mathematics is intended to give the students deep understanding of the principles of Mathematical sciences while expanding their knowledge in the allied areas through elective courses. The curriculum has been designed so as to prepare the students to take up a research career either in academia or in industries on completion of the program. The students will be equally equipped to take up professional career in Industries.

The structure of the proposed program has been designed by providing a balance among theory, application, and research components. The program aims to equip students with the necessary theoretical knowledge, analytical skills, and problem-solving techniques required to excel in the field of mathematics. The program is designed in such a way that students will have enough choices to learn their desired subjects by taking number of elective courses from and outside of the discipline. The curriculum focuses on an interdisciplinary approach wherein students learn theory and its applications through fundamental core courses.

Tutorials/assignments have been prescribed in each course. It is important to develop among the students an ability to understand and apply the theoretical concepts. The teachers can attempt to accomplish this task during tutorials. Thought provoking exercises along with routine ones may be given to students for assignments. It is advisable to form groups in a batch and the same tutorial problems be conducted using different methods if applicable to get the feel of the solution space for solving problems.

This syllabus should motivate the students and make them appreciate the beauty and value of Mathematics.

M.Sc. (Mathematics) Semester I

Course	Course Code	Course Name	Credits	Teaching Scheme				Examination Scheme						
								Theory				Practical		
				Lecture Per Week	Practical per Week	Total Periods	Duration in Hours	Max. Marks (Theory)	Max. Marks (Internals)	Total Marks	Minimum Passing Marks	Max Marks	Total Marks	Minimum Passing Marks
DSC1	MSMTDC101T	Linear Algebra	4	4	--	4	4	60	40	100	40	--	--	--
DSC2	MSMTDC102T	Real and Complex Analysis	4	4	--	4	4	60	40	100	40	--	--	--
DSC3	MSMTDC103T	Differential Calculus	2	2	--	2	2	30	20	50	20	--	--	--
DSC4	MSMTDC104T	Ordinary Differential Equations	2	2	--	2	2	30	20	50	20	--	--	--
DSE1 (Any One)	MSMTDE101T	Probability Theory	4	4	--	4	4	60	40	100	40	--	--	--
	MSMTDE102T	Integral Transforms	4	4	--	4	4	60	40	100	40	--	--	--
RM	MSMTRM101T	Research Methodology	4	4	--	4	4	60	40	100	40	--	--	--
DSE1 (Any One)	MSMTDE101P	Probability Theory LAB	2	--	8	8	4	--	--	--	--	50	50	20
	MSMTDE102P	Integral Transforms LAB	2	--	8	8	4	--	--	--	--	50	50	20

M.Sc. (Mathematics) Syllabus

Semester I

MSMTDC101T Linear Algebra

Course Credit: 04 (Credit: Th(4)) **Total Contact Hours: 60 Hrs.**

Unit I: Vector Space and Linear Transformations (15 Hours)

Review of Vector Spaces over a field F, Subspaces, Basis and dimensions.

Linear Transformations, rank and nullity of a linear transformation, matrix associated with a linear transformation, rank of a matrix and its equivalence to rank of T.

Linear functional, dual space, dual basis, double dual, canonical isomorphism of a vector space and its double dual, transpose of a linear transformation.

Unit II: Determinants and Eigenvalues (15 Hours)

Multilinear maps, determinant function, uniqueness of determinant, properties of determinant.

Characteristic polynomial, annihilating polynomial, Cayley Hamilton Theorem, minimal polynomial, eigenvalues and eigenvectors.

Unit III: Elementary Canonical Forms and Inner Product Space (15 Hours)

Invariant subspaces, diagonalization, triangularization

Jordan Canonical form (only examples)

Application: Singular Value Decomposition (SVD).

Inner Product Spaces, Norm induced by inner product, Cauchy Schwartz inequality, Orthogonal Bases, Gram-Schmidt Orthogonalization.

Unit III: Operators and Bilinear Forms (15 Hours)

Adjoint of an operator, Unitary operators, Self-adjoint and normal operators, invertible operators, nilpotent operators.

Bilinear forms, Symmetric bilinear forms, Classification.

References

1. Kenneth Hoffman and Ray Kunze: Linear Algebra, Prentice Hall.
2. S. Kumaresan: Linear Algebra – A Geometric Approach, Prentice Hall India; Carl D. Mayer
3. Serge Lang: Linear Algebra, Springer-Verlag Undergraduate Text in Mathematics.
4. I. N. Herstein, Topics in Algebr, Macmillan, Indian Edition.
5. N.S. Gopalkrishnan: University Algebra, New Age International, third edition, 2015.
6. Michael Artin: Algebra, Prentice-Hall India.

MSMTDC102T Real and Complex Analysis

Course Credit: 04 (Credit: Th(4)) **Total Contact Hours: 60 Hrs.**

Unit I: Topology of $\mathbb{R}^n$ (15 Hours)

Review of Metric space, open set, closed set, interior of a set, closure of a set, limit point, boundary of a set.

Compact sets, Heine-Borel property, Bolzano-Weierstrass Theorem, Continuity, images of Compact and Connected sets, boundedness of continuous functions on compact sets.

Pointwise and uniform convergence of sequence and series of real valued functions, The Weierstrass M-test, integration and differentiation of series, space of continuous functions.

Unit II: Holomorphic functions (15 Hours)

Review of Complex Numbers. Geometry of the complex plane. Riemann sphere. Complex sequences and series. Sequences and series of functions in $\mathbb{C}$. Uniform convergence.

Ratio test and root test for convergence of a series of complex numbers. Complex Power series. Radius of convergence of a power series. Cauchy-Hadamard formula. Analytic functions. Examples of convergent power series such as e^z, $\cos z$, $\sin z$ and their basic properties. The logarithm as the inverse of exponential. Complex differentiable (Holomorphic) functions, Cauchy-Riemann equations.

Unit III: Contour integration, Cauchy-Gursat theorem (15 Hours)

Integration on Curves, Cauchy-Goursat Theorem for a rectangular region or a triangular region. Primitives, Existence of primitives. Contour integration, Local Cauchy's Formula for discs. Cauchy's theorem (homotopy version or homology version), The index (winding number) of a closed curve, Cauchy integral formula, Cauchy's estimates, Power series representation of holomorphic functions. Liouville's theorem, The Fundamental theorem of Algebra. Morera's theorem.

Unit IV: Residue Calculus (15 Hours)

Counting zeros of a holomorphic function. Identity principle. Meromorphic functions, singularities: Zeros and Poles, Isolated singularities: removable singularities and Removable singularity theorem, essential singularities. Laurent Series development. Casorati-Weierstrass's theorem. Residue Theorem and evaluation of standard types of integrals by the residue calculus method. Argument principle. Rouché's theorem. Open Mapping Theorem, Maximum modulus theorem. Entire functions.

References

1. J. B. Conway, Functions of one Complex variable, Springer.
2. L. V. Ahlfors: Complex analysis, McGraw Hill.
3. A. R. Shastri: An introduction to complex analysis, Macmillan.
4. Serge Lang: Complex Analysis, Springer.
5. R. V. Churchil, J. W. Brown: Complex Variables and Applications, McGraw Hill.
6. R. Remmert: Theory of complex functions, Springer.
7. S. Ponnusamy, Herb Silverman: Complex Variables with Applications, Birkhauser Boston - Basel - Berlin.

MSMTDC103T Differential Calculus

Course Credit: 02 (Credit: Th(2)) **Total Contact Hours: 30 Hrs.**

Unit I: Differentiable Mapping (15 Hours)

The definition of the derivative, matrix representation, continuity of differentiable mapping, conditions of differentiability, chain rule, product rule and gradients, mean-value theorem, Taylor's theorem and higher derivatives, maxima and minima.

Unit II: Inverse and Implicit Function Theorems (15 Hours)

Inverse function theorem, implicit function theorem, consequences of implicit function theorem.

References:

1. Jerrold E. Marsden: Elementary Classical Analysis, W.H. Freeman and Company, San Fancisco.
2. S. Kumaresan: Topology of Metric spaces, Alpha Science International Ltd. Harrow, U.K.
3. Robert G. Bartle: Elements of Real analysis, John Wiley & Sons, Inc., New York.
4. Tom M. Apostol: Mathematical Analysis, Addison Wesley.
5. Walter Rudin: Principles of mathematical analysis, McGraw-Hill.

MSMTDC104T Ordinary Differential Equations

Course Credit: 02 (Credit: Th(2)) **Total Contact Hours: 30 Hrs.**

Unit I: Picards Theorem (15 Hours)

Existence and Uniqueness of solutions to initial value problem of first order ODE. Picard's scheme of successive Approximations, Lipschitz condition, convergence of the successive approximation.

Existence and uniqueness results for an n-th order linear ODE with constant coefficients and variable coefficients.

Unit II: Ordinary Differential Equation (15 Hours)

Linear dependence and independence of solutions of a homogeneous n-th order linear ODE, Wronskian matrix, Lagrange's Method (variation of parameters), algebraic properties of the space of solutions of a non-homogeneous n-th order linear ODE.

System of first order linear ODE with constant coefficients and variable coefficients, reduction of an n-th order linear ODE to a system of first order ODE.

Sturm-Liouville Theory: Sturm-Liouville Separation and comparison Theorems, Oscillation properties of solutions, Eigenvalues and eigenfunctions of Sturm-Liouville Boundary Value Problem.

References

(1) E.A. Codington, N. Levinson: Theory of ordinary differential Equations, Tata McGraw-Hill, India.

(2) D. Somasundaram: Ordinary Differential Equations, Narosa Publications.

(3) G.F. Simmons: Differential equations with applications and historical notes, McGraw-Hill international edition.

(4) Morris W. Hirsch and Stephen Smale: Differential Equations, Dynamical Systems, Linear Algebra, Elsevier.

MSMTDE101T Probability Theory

Course Credit: 06 (Credit: Th(4) + Pr(2)) **Total Contact Hours: 60 Hrs.**

Unit I: Probability basics (15 Hours)

Modelling Random Experiments: Introduction to probability, probability space, events.

Classical probability spaces: uniform probability measure, fields, finite fields, finitely additive probability, Inclusion-exclusion principle, σ-fields, σ-fields generated by a family of sets, σ-field of Borel sets, Limit superior and limit inferior for a sequence of events.

Unit II: Probability measure (15 Hours)

Probability measure, Continuity of probabilities, First Borel-Cantelli lemma, Discussion of Lebesgue measure on σ-field of Borel subsets of assuming its existence, Discussion of Lebesgue integral for non-negative Borel functions assuming its construction.

Discrete and absolutely continuous probability measures, conditional probability, total probability formula, Bayes formula, Independent events.

Unit III: Random Variables (15 Hours)

Random variables, simple random variables, discrete and absolutely continuous random variables, distribution of a random variable, distribution function of a random variable, Bernoulli, Binomial, Poisson and Normal distributions, Independent random variables, Expectation and variance of random variables both discrete and absolutely continuous.

Unit IV: Limit Theorems (15 Hours)

Conditional expectations and their properties, characteristic functions, examples, Higher moments examples, Chebyshev inequality, Weak law of large numbers, Convergence of random variables, Kolmogorov strong law of large numbers (statement only), Characteristics function and properties, Central limit theorem (statement only).

References

1. M. Capinski, Tomasz Zastawniak: Probability Through Problems.

2. B. R. Bhat: Modern Probability Theory (An Introductory Textbook), New Age International Publishers.

3. J. F. Rosenthal: A First Look at Rigorous Probability Theory, World Scientific.

4. Kai Lai Chung, Farid AitSahlia: Elementary Probability Theory, Springer Verlag.
5. Sheldon Ross, A First Course in Probability, Prentice Hall India.
6. Vijay K. Rohatgi, A. K. Md. Ehsanes Saleh, An Introduction to Probability and Statistics, second edition, Wiley series.

MSMTDE102T Integral Transforms

Course Credit: 06 (Credit: Th(4) + Pr(2)) **Total Contact Hours: 60 Hrs.**

Unit I: Laplace Transform (15 Hours)

Definition of Laplace Transform, Laplace transforms of some elementary functions, Properties of Laplace transform, Laplace transform of the derivative of a function, Inverse Laplace Transform, Properties of Inverse Laplace Transform, Inverse Laplace Transform of derivatives, Convolution Theorem, Heaviside's expansion theorem, Application of Laplace transform to solutions of ODEs and PDEs.

Unit II: Fourier Transform (15 Hours)

Fourier Integral theorem, Properties of Fourier Transform, Inverse Fourier Transform, Convolution Theorem, Fourier Transform of the derivatives of functions, Parseval's Identity, Relationship of Fourier and Laplace Transform, Application of Fourier transforms to the solution of initial and boundary value problems.

Unit III: Mellin Transform (15 Hours)

Properties and evaluation of Mellin transforms, Convolution theorem for Mellin transform, Complex variable method and applications.

Unit IV: Z-Transform (15 Hours)

Definition of Z-transform, Inversion of the Z-transform, Solutions of difference equations using Z-transform.

References

1. Brian Davies, Integral transforms and their Applications, Springer.
2. L. Andrews and B. Shivamogg, Integral Transforms for Engineers,Prentice Hall of India.
3. I.N.Sneddon, Use of Integral Transforms, Tata-McGraw Hill.
4. R. Bracemell, Fourier Transform and its Applications, MacDraw hill.

MSMTRM101T Research Methodology

Course Credit: 04 (Credit: Th(4)) **Total Contact Hours: 60 Hrs.**

Unit I: Inferential Statistics (15 Hours)

Review: Measures of Central tendencies: Mean, Median, Mode, Range, Mean deviation, standard deviation (no questions to be asked).

Sampling and Sampling Distribution, sampling methods, sampling errors.

Point and interval estimation, confidence interval, formulation of statistical hypothesis, Type I and Type II errors, Inference about a population variance.

Chi-square test of independence, Tests of Goodness of Fit and independence, ANOVA.

Unit II: Sampling Distributions (15 Hours)

Introduction to Sampling distributions, Student's t distribution, Chi square distribution, Snedecor's F distribution, Interrelations among t, chi-square and F distribution.

Unit III: Estimation & Testing of hypothesis (15 Hours)

Estimator, Estimate, Methods of point estimation – Maximum Likelihood (ML) method (the asymptotic properties of ML estimators are not included), Large sample properties of ML estimator (without proof) - applications, Method of moments, method of least squares, method of minimum chi square, Introduction of Interval Estimation, Confidence limits and confidence coefficient.

Definitions: population, statistic, parameter, standard error of estimator, Concept of null hypothesis and alternative hypothesis, critical region, level of significance, one sided and two-sided tests, p value.

Unit IV: Regression Analysis (15 Hours)

Simple linear regression, least square method, coefficient of determination, Using the Estimated Regression Equation for Estimation and Prediction. Multiple regression, Least Squares Method, Multiple Coefficient of Determination, Regression Analysis

References

1. G. Casella and R. L. Berger. Statistical Inference. Duxbury Press.

2. A. M. Mood, F. A. Graybill and D. C. Boes, Introduction to The Theory of Statistics, Third Edition, Mc Graw Hill Education.

3. Laura Chihara and Tim Hesterberg, Mathematical Statistics and Resampling and R.John Wiley & Sons.

4. L. Wasserman, All of Statistics, Springer.

5. Daniel Sabanés Bové and Leonhard Held, Applied Statistical Inference: Likelihood and Bayes, Springer

6. Anderson, Sweeney, Williams: Statistics for Business and Economics, South-Western Cengage Learning

7. Douglas C. Montgomery, Elizabeth A. Peck, G. Geoffrey Vining: Introduction to Linear Regression Analysis, Wiley Publication.

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Proposed M. Sc. (Mathematics) Syllabus

Semester II

With Effect from Academic Year 2023 - 2024

M.Sc. (Mathematics) Semester II

Course	Course Code	Course Name	Credits	Teaching Scheme				Examination Scheme						
								Theory				Practical		
				Lecture Per Week	Practical per Week	Total Periods	Duration in Hours	Max. Marks (Theory)	Max. Marks (Internals)	Total Marks	Minimum Passing Marks	Max Marks	Total Marks	Minimum Passing Marks
DSC5	MSMTDC201T	Group Theory	4	4	--	4	4	60	40	100	40	--	--	--
DSC6	MSMTDC202T	Topology	4	4	--	4	4	60	40	100	40	--	--	--
DSC7	MSMTDC203T	Optimization Techniques	2	2	--	2	2	30	20	50	20	--	--	--
DSC8	MSMTDC204T	Partial Differential Equations	2	2	--	2	2	30	20	50	20	--	--	--
DSE2 (Any One)	MSMTDE201T	Measure Theory and Integration	4	4	--	4	4	60	40	100	40	--	--	--
	MSMTDE202T	Differential Geometry	4	4	--	4	4	60	40	100	40	--	--	--
DSE2 (Any One)	MSMTDE201P	Measure Theory and Integration LAB	2	--	8	8	4	--	--	--	--	50	50	20
	MSMTDE202P	Differential Geometry LAB	2	--	8	8	4	--	--	--	--	50	50	20
FP/OJT	MSMTFP201T/ MSMTOJ201P	Field Project/OJT	4	4	--	4	4	60	40	100	40	--	--	--

M.Sc. (Mathematics) Syllabus

Semester II

MSMTDC201T Group Theory

Course Credit: 04 (Credit: Th(4)) **Total Contact Hours: 60 Hrs.**

Unit I: Groups, Structure Theorem (15 Hours)

Review: Groups, subgroups, normal subgroups. Group homomorphism, kernel of a homomorphism. Cyclic groups. Lagranges theorem. Examples of groups such as Permutation groups, Dihedral groups, Matrix groups, the group of units of $\mathbb{Z}_n$, Quotient groups. First, Second and third isomorphism theorems for groups. (no questions to be asked).

Automorphisms of a group. If G is a group, then $Aut(G)$, the set of all automorphisms of G, is a group under composition.

Direct product of groups - external direct product, Internal direct product, Semi-direct products.

Structure theorem of finite abelian groups and applications.

Unit II: Groups action (15 Hours)

Groups acting on sets, normalizer $N(a)$ of an element $a \in G$, conjugacy class $C(a)$ of a in G. Class equation. Applications such as:

(1) If G is a group of order p^n where p is a prime number, then $Z(G) \neq \{e\}$.

(2) Any group of order p^2, where p is a prime number, is abelian.

(3) Burnside lemma and Application

Unit III: Sylow Theorems (15 Hours)

Cauchys theorem, p-groups, Sylow's theorems and applications. Classification of groups of order up to 15.

Unit IV: Solvable and Nilpotent Groups (15 Hours)

Simple groups, A5 is simple.

Composition Series, Normal series, Zassenhaus Lemma, Jordan-Holder theorem.

Solvable groups, Solvability of all groups of order less than 60, Nilpotent groups.

References

(1) M. Artin: Algebra, Prentice Hall of India.

(2) D.S. Dummit and R.M. Foote: Abstract Algebra, John Wiley and Sons.

(3) I.S. Luthar and I.B.S. Passi: Algebra (Volume 1) Groups (Narosa Publishing House

(4) I. N. Herstein: Topics in Algebra, Wiley-India.

(5) Thomas W. Hungerford: Abstract Algebra: An Introduction; BROOKS/COLE, CENGAGE Learning

(6) Hungerford, Thomas W: Algebra, Springer

(7) John B. Fraleigh: A First Course in Abstract Algebra, Pearson new international edition.

(8) A. Tucker: Applied Combinatorics, John Wiley & Sons.

MSMTDC202T Topology

Course Credit: 04 (Credit: Th(4)) **Total Contact Hours: 60 Hrs.**

Unit I: Topological Spaces and Continuous Functions (15 Hours)

Topological Spaces, Basis for Topology, The Order Topology, The product Topology, The Subspace Topology , Closed Sets and Limit Points , Continuous functions, Quotient Topology.

Unit II: Connectedness and Compactness(15 Hours)

Connected Spaces, Connected Subspace on Real Line. Compact Spaces, Compact Subspace on the Real Line, Components and Local Connectedness, Limit Point Compactness, Local Compactness.

Unit III: Countable Axioms (15 Hours)

First countable, Second Countable, Separable, Lindelof, Separation Axioms: Regular and Normal spaces.

Unit IV: Tychonoff Theorem (15 Hours)

The Urysohn's Lemma, The Urysohan Metrization Theorem and, The Tietze extension theorem (statement only). the Tychonoff Theorem.

References

(1) James Munkres: Topology, Pearson.

(2) George Simmons: Topology and Modern Analysis, Tata Mcgraw-Hill.

(3) J. Dugundji : Topology (Allyn and Bacon, Boston, 1966.)

(4) K. D. Joshi : Introduction to General Topology (Wiley Eastern Limited).

(5) Stephen Willard, General Topology, Addison-Wesley Publishing Company, 1970

(6) Sheldon W. Davis, Topology (The Walter Rudin Student Series in Advanced Mathematics), TATA McGraw-Hill.2006.

(7) J. L. Kelley : General Topology (Springer Verlag, New York 1991.)

(8) M. A. Armstrong: Basic Topology, Springer UTM.

MSMTDC203T Optimization Techniques

Course Credit: 02 **Total Contact Hours: 30 Hrs.**

Unit I: Unconstrained Optimization (15 Hours)

Introduction to Optimization - Overview of optimization problems, Types of optimization problems, Optimization modelling.

Unconstrained Optimization - One-dimensional optimization methods, Multidimensional optimization methods, Gradient-based methods, Newton's method, Convergence analysis

Unit II: Constrained Optimization (15 Hours)

Equality and inequality constraints, Karush-Kuhn-Tucker (KKT) conditions, Lagrange multipliers, Penalty and barrier methods, Sequential quadratic programming.

Formulating linear programming problems, Simplex method, Duality theory.

References Books

(1) Jorge Nocedal and Stephen J. Wright, Numerical Optimization

(2) Stephen Boyd and Lieven Vandenberghe, Convex Optimization

(3) Dimitris Bertsimas and John N. Tsitsiklis, Introduction to Linear Optimization.

MSMTDC204T Partial Differential Equations

Course Credit: 02 **Total Contact Hours: 30 Hrs.**

Unit I: First and Second order Partial Differential Equations (15 Hours)

Review of Multivariable Calculus,Essential Ordinary Differential Equations, Integral Curves and Surfaces of Vector Fields,Solving Equations of the form: $dx/P = dy/Q = dz/R$.

First-Order Partial Differential Equations(PDEs)– Formation and classification of first-order PDEs, Linear and Quasilinear first-order PDEs, Cauchy's problem for first order PDEs, The Cauchy Kowalevski Theorem, Integral surfaces passing through a given curve, Nonlinear first-order PDEs, The method of characteristics, Compatible systems, Charpit's method, Jacobi's method for nonlinear PDEs.

Second-Order PDEs - Classification, Canonical forms, Well-posed problems,Superposition principle.

Unit III: The Wave and Laplace's Equation (15 Hours)

Wave Equation - Derivation of the wave equation, The infinite string problem, The D'Alembert solution of the wave equation, The semi-infinite string problem, The finite vibrating string problem, The method of separation variables, The inhomogeneous wave equation.

Laplace's Equation – Basic concepts, Types of boundary value problems, The maximum and minimum principle, Green's identity and fundamental solution.

References

(1) D.Bleecker and G. Csordas,Basic Partial Differential Equations,Van Nostrand Reinhold, New York, 1992.

(2) C. Constanda,Solution Techniques for Elementary Partial Differential Equations, Chapman & Hall/CRC, New York, 2002.

(3) S. J. Farlow,Partial Differential Equations for Scientists and Engineers,Birkh auser, New York, 1993.

(4) F. John,Partial Differential Equations,Springer-Verlag, New York, 1982.

(5) E.Kreyszig,Advanced Engineering Mathematics, Wiley, 2011.

(6) J. Marsden and A. Weinstein, Calculus III,Springer-Verlag, New York, 1985.

MSMTDE201T Measure Theory and Integration

Course Credit: 04 (Credit: Th(4)) **Total Contact Hours: 60 Hrs.**

Unit I: Measures (15 Hours)

Algebra $\mathcal{A}$ of sets, σ-algebra $\mathcal{M}$, Borel σ-algebra $\mathcal{B}_X$, Product algebra.

Measure μ and its properties, Outer measure μ^* on subsets of X, Carathéodory construction of measure, measure space $(X, \mathcal{M}, \mu)$, Inner measure μ_*. Measurability criteria.

Borel measures on $\mathbb{R}$, Lebesgue-Stieltjes measure μ_F associated with an increasing right-continuous $F : \mathbb{R} \to \mathbb{R}$, Lebesgue measure m on $\mathbb{R}$ and on $\mathbb{R}^n$, Cantor sets in $\mathbb{R}$, construction of a non-measurable set in $\mathbb{R}$.

Unit II: Measurable functions and Integration (15 Hours)

Measurable functions on $(X, \mathcal{M}, \mu)$, Simple functions, properties of measurable functions, Integration of nonnegative functions, Monotone Convergence Theorem, Fatou's Lemma,

Unit III: Integration of Complex Functions (15 Hours)

Integration of complex functions, Dominated Convergence Theorem, modes of convergence, Lusin's theorem, Egorov's theorem. The classes $L^+(\mu), L^1(\mu), \mathcal{L}^1(\mu), L^p(\mu), \mathcal{L}^p(\mu)$, approximation by functions with compact support.

Unit IV: The Lebesgue Integral on $\mathbb{R}^n$ (15 Hours)

Product measures, Monotone Class Lemma, Integration on product spaces, Fubini-Tonelli Theorems, Lebesgue measure on $\mathbb{R}^n$, behavior of Lebesgue integral under linear transformations.

References

1. Andrew Browder, Mathematical Analysis: An Introduction, Springer Undergraduate Texts in Mathematics.
2. H. L. Royden, Real Analysis, PHI.
3. W. Rudin, Real and Complex Analysis, McGraw-Hill India.
4. G. B. Folland, Real Analysis: Modern Techniques and Their Applications, Wiley-Interscience.
5. M. Capinski and E. Kopp, Measure, Integral and Probability, Springer.

MSMTDE202T Differential Geometry

Course Credit: 04 **Total Contact Hours: 60 Hrs.**

Unit I: Isometries of $\mathbb{R}^n$ (15 Hours)

Orthogonal transformations of $\mathbb{R}^n$ and Orthogonal matrices. Any isometry of $\mathbb{R}^n$ fixing the origin is an orthogonal transformation. Any isometry of $\mathbb{R}^n$ is the composition of an orthogonal transformation and a translation. Orientation preserving isometries of $\mathbb{R}^n$.

Reflection map about a hyperplane W of $\mathbb{R}^n$ through the origin: Let W be a vector subspace of $\mathbb{R}^n$ of dimension $n-1$. Let n be any unit vector in $\mathbb{R}^n$ orthogonal to W. Define $T(v) = v - 2\langle v, n\rangle n, (v \in \mathbb{R}^n)$. Then T is an orthogonal transformation of $\mathbb{R}^n$, and T is independent of the choice of n. Any isometry of $\mathbb{R}^n$ is the composition of at most $n+1$ many reflections.

Isometries of the plane: Rotation map of $\mathbb{R}^2$ about any point p of $\mathbb{R}^2$, reflection map of $\mathbb{R}^2$ about any line ℓ of $\mathbb{R}^2$. Glide reflection of $\mathbb{R}^2$ (obtained by reflecting about a line ℓ and then translating by a non-zero vector v parallel to ℓ). Any isometry of $\mathbb{R}^2$ is a rotation, a reflection, a glide reflection, or the identity.

Unit II: Curves (15 Hours)

Regular curves in $\mathbb{R}^2$ and $\mathbb{R}^3$, Arc length parametrization, Signed curvature for plane curves, Curvature and torsion of curves in $\mathbb{R}^3$ and their invariance under orientation preserving isometries of $\mathbb{R}^3$. Serret-Frenet equations. Fundamental theorem for space curves in $\mathbb{R}^3$.

Unit III: Regular Surfaces (15 Hours)

Regular surfaces in $\mathbb{R}^3$, Examples. Surfaces as level sets, Surfaces as graphs, Surfaces of revolution. Tangent space to a surface at a point, Equivalent definitions. Smooth functions on a surface, Differential of a smooth function defined on a surface. Orientable surfaces. Mobius band is not orientable.

Unit IV: Curvature (15 Hours)

The first fundamental form. The Gauss map, the shape operator of a surface at a point, self-adjointness of the shape operator, the second fundamental form, Principle curvatures and directions, Euler's formula, Meusnier's Theorem, Normal curvature. Gaussian curvature and mean curvature, Computation of Gaussian curvature, Isometries of surfaces, Covariant differentiation, Gauss's Theorema Egragium (statement only), Geodesics.

References Books

(1) S. Kumaresan, A Course in Riemannian geometry

(2) M. Artin, Algebra, Prentice Hall of India.

(3) M. DoCarmo, Differential geometry of curves and surfaces, Dover.

(4) C. Bar, Elementary Differential geometry, Cambridge University Press, 2010.

(5) A. Pressley,, Elementary Differential Geometry, Springer UTM.

MSMTFP201P/MSMTOJ201P Field Project/On Job Training

Course Credit: 04 **Total Contact Hours: 60 Hrs.**

Evaluation will be based on Dissertation submitted and presentation in front of the internal examiner (other than the Guide) within the department and external examiner.

Each students should submit the monthly report of their project to the department.

Internal Marks - 40 (to be given by the Guide)

External Examination - Project content & Project Report (20 marks) + Project Presentation (20 marks) + Viva (20 marks)